Meriam 3/7: Potential Energy

Recap: $T_2 = T_1 + U_{1\to 2}$, where

- (a) $U_{gravity} = -mg\Delta h = mgh_1 - mgh_2$
- (b) $U_{kinetic\ friction} = -F_{friction}\int ds$
- (c) $U_{elastic} = -\frac{1}{2}k(x_2^2 - x_1^2)$

Key Point: U_g & $U_{elastic}$ depend only on end points of trip, but $U_{friction}$ is path dependent

Define Conservative Force: The work done by a conservative force is independent of path

Other conservative Forces?

Define Potential Energy = V = the amount of work done by a conservative force acting on a mass moving from an initial position (1) to a reference position (2).

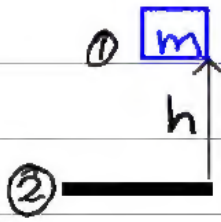
- Potential energy is only defined for conservative forces
- Potential energy depends on choice of reference position

- Since selection of a reference location for V is arbitrary, select a convenient location for which arithmetic is simple.

Eg: A mass m is located at a height h above the ground. Find V_g.

Answer ① Select ground level as our reference

→ V_g = work done by gravity as the mass moves from original location to reference



→ $V_g = -mg\Delta h = -mg(h_2 - h) = mgh$ ($h_2 = 0$)

Answer ② Select current position of the mass as the reference point:

→ $V_g = -mg\Delta h = -mg(h_2 - h) = 0$ ($h_2 = h$)

Answer ③ Select sealevel as the reference point

→ $V_g = mgh_s$, h_s = height above sea level

Often we pick ground level or sea level as a reference point that everyone is familiar with.

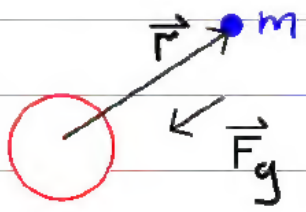
→ $\boxed{V_g = mgh}$

For work done by gravity over long distances in outer space, $g \neq$ constant.

→ Use Newton's law of gravity to find work done on a mass as it moves from r to r_2 wrt earth:

$$V_g = \int_r^{r_2} -\frac{G m_e m}{(r')^2}\, dr' = \frac{G m_e m}{r'}\Bigg|_{r'=r}^{r'=r_2}$$

select reference distance $r_2 = \infty$ to keep arithmetic simple:



→ $$V_g = -\frac{G m_e m}{r}$$

Springs: It is arithmetically convenient to select the relaxed position of a spring as the reference point:

→ $V_e = -\frac{1}{2} k(x_2^2 - x^2)$, where x = spring extension

→ $\boxed{V_e = \frac{1}{2} k x^2}$ (+/− sign doesn't matter)

Total potential energy of mass m:

$$V = V_g + V_e + \cdots$$

Note: $(U_g)_{1\to 2} = -mg\Delta h = mgh_1 - mgh_2$

$= V_{g,1} - V_{g,2}$

$$\rightarrow \quad U_g = -\Delta V_g$$

Similarly,

$$U_e = \tfrac{1}{2} k x_1^2 - \tfrac{1}{2} k x_2^2 = V_{e,1} - V_{e,2}$$

$$\rightarrow \quad U_e = -\Delta V_e$$

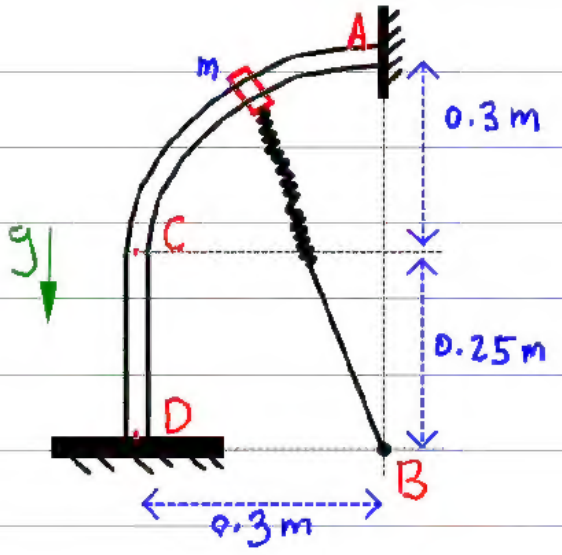
In general, $U_{1\to 2} = -\Delta V$

or $T_2 = T_1 + U_{1\to 2} \rightarrow -\Delta V$

$$\rightarrow \quad T_1 + V_1 = T_2 + V_2$$, if the work is done by conservative forces. Energy may change its form, but total energy is conserved

Eg.

A "slider" can move along a bar between points A and D



Given:

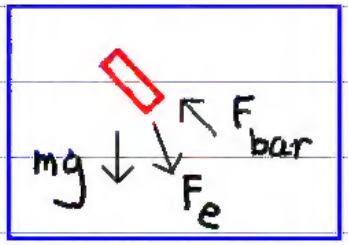
$m = 2.55$ kg

$k = 400$ N/m

initial $V_A = 1.5$ m/s ←

friction = 0

relaxed spring length = 0.3 m



(a) Find v of slider as it approaches point "C"

Answer:

$$T_A + V_A = T_C + V_C \quad \text{or} \quad T_C = T_A + U_{A \to C}$$

T_A:

$$T_A = \tfrac{1}{2} m V_A^2 = (\tfrac{1}{2})(2.55)(1.5)^2 = 2.869 \text{ Joules}$$

V_A: Define C as reference height

$$V_A = (V_g)_A + (V_e)_A = mgh_A + \tfrac{1}{2}kx^2$$

$$V_A = (2.55)(9.81)(0.3) + (\tfrac{1}{2})(400)(0.55 - 0.3)^2$$

$$V_A = 20 \text{ Joules}$$

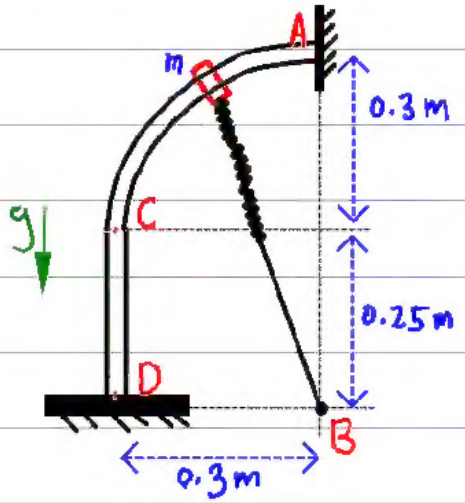
V_C:

$$V_C = (V_g)_C + (V_e)_C$$

$$= 0 + \tfrac{1}{2}k(\overline{BC} - 0.3)^2$$

$$= (\tfrac{1}{2})(400)(\underbrace{\sqrt{0.25^2 + 0.3^2}}_{0.3905} - 0.3)^2$$

$$V_C = 1.64 \text{ J}$$



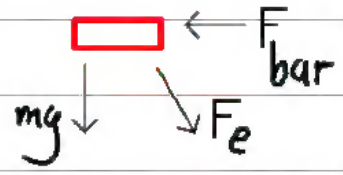
$$T_A + V_A = T_C + V_C$$

$$2.869 + 20 = (\tfrac{1}{2})(2.55)(V_C)^2 + 1.64$$

$$\boxed{V_C = 4.08 \text{ m/s}}$$

(Answer depends on $\Delta V_g = (V_g)_C - (V_g)_A$, but does not depend on choice of reference altitude.)

(b) Find force $\vec{F}_B$ of bar on slider as it approaches "C".



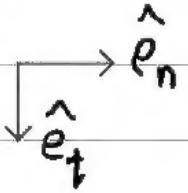
$$\Sigma F_n = m a_n$$

$$-F_{bar} + F\left(\frac{0.3}{BC}\right) = m\frac{v_c^2}{\rho}$$

$$-F_{bar} + (400)(0.3905 - 0.3)\left(\frac{0.3}{0.3905}\right) = 2.55\left(\frac{4.08^2}{0.3}\right)$$

$$\boxed{F_{bar} = -113.7 \text{ Newtons}}$$

Direction?



Summary

- Definitions of: Conservative Force
  Potential Energy
- For a conservative force, $U_{1\to 2} = -\Delta V$
- For a particle moving from point 1 to 2,

$$T_2 = T_1 + U_{1\to 2} = T_1 - \Delta V$$

$$\rightarrow T_2 + V_2 = T_1 + V_1$$

For next lecture

- Read Meriam 3/8 — 3/9: Linear Momentum